

## Comments on domain walls in holographic duals of mass deformed conformal field theories

Akikazu Hashimoto

Department of Physics, University of Wisconsin, Madison, WI 53706, USA

### Abstract

We consider M-theory backgrounds which are gravity duals of mass deformed superconformal field theories in 2+1 dimensions. The specific examples we consider are the  $B_8$ , Stenzel, and the Lin-Lunin-Maldacena geometries. These geometries contain compact 4-cycles on which one can wrap an M5-brane to create an object which behaves effectively like a domain wall in 2+1 dimensions. We review the quantization of flux and charges of these M-theory backgrounds, and confirm that the back reaction of the domain wall shifts the charges in a manner consistent with these quantization conditions, paying particular attention to various subtle half integer shifts of the charge lattice which arise as a part of the complete story. We also describe a configuration of a stationary, merging M2/anti M2 pair in the Lin-Lunin-Maldacena background, which can also be interpreted as a domain wall, and compare its basic properties with the expectations from its field theory description.

# 1 Introduction

Field theories in 2+1 dimensions are useful laboratories for exploring dynamical issues in a framework that is well behaved in the ultra-violet. Unlike in 3+1 dimensions, gauge field theories in 2+1 dimensions are super-renormalizable regardless of the number of charged matter fields, and the duality cascades do not continue indefinitely as one flows to the UV.

For some field theories, holography provides some additional tools to probe their dynamical features. The prototype holographic duality for field theories in 2+1 dimensions is the duality of ABJM [1] which relates  $U(N)_k \times U(N)_{-k}$  Chern-Simons theory with bi-fundamental matter fields and a specific superpotential to  $AdS_4 \times S^7/Z_k$  geometry. This duality can be understood as relating the decoupling limit of the world volume theory on an M2 brane placed in the  $R^8/Z_k$  transverse space to its gravity description in the near horizon limit.

The ABJM duality has been generalized in a variety of ways including changing the gauge group and the matter content, deforming the IR, and scaling in new physics in the UV such as the Yang-Mills interaction. In order to make the holographic duality precise, it is important to correctly identify and relate the discrete and continuous parameters appearing on both sides of the duality. Identification of discrete parameters generally involve understanding quantized fluxes and charges on the gravity side of the correspondence.

As we have seen in a number of examples, the task of quantifying and discretizing fluxes and charges involves some subtleties. This stems from the fact that there are several notions of charges, which in simple contexts are indistinguishable, but can take on distinct values and behave differently in more general settings. To avoid the potential pitfall of confusing these subtle notions of charges, we follow [2] and use different names, brane, bulk, Page, and Maxwell, in order to distinguish between them. These charges take on different quantitative values in the presence of torsion and fluxes when the space-time theory contain Chern-Simons terms, as is often the case in supergravity theories. Page charges are integer quantized but not gauge invariant. As such they map naturally to discrete parameters such as level and rank which also exhibit ambiguity on the field theory side via duality relations. Maxwell, brane, and bulk charges are continuous and encode parameters and observables of the theory. A useful overview of the subtle roles of these charges can be found in [3]. One interesting outcome of the analysis of charges for the ABJM theory is the prediction of a phase diagram in charge space

$$N - \frac{l(l-k)}{2k} > 0 \tag{1.1}$$

for the existence of a superconformal fixed point for the  $U(N)_k \times U(N+l)_{-k}$  theory. It implies that the Yang-Mills-Chern-Simons-Matter theory [3,4] (or some other UV embedding

of the Chern-Simons-Matter theory) with gauge group and level violating this inequality must exhibit drastically different low energy effective dynamics with spontaneously broken supersymmetry. The precise nature of the low energy effective dynamics of this phase is not currently well known, although there have been some attempts to investigate these issues [5, 6].

In this article, we examine the issues which arise in gauge/gravity correspondences when the superconformal field theories are mass deformed in the IR. On the gravity side, these deformations generally takes a background in a form of a cone such as the  $R^8/Z_k$  and blows up the point on the tip into a 4-cycle. We will focus on the class of geometries whose supergravity solution are known explicitly: the  $B_8$  geometry [7], the Stenzel geometry [8], and the LLM geometry [9]. For the LLM geometry, the dual field theory candidate is known very explicitly [10]. For the  $B_8$  and Stenzel, the field theory dual is not as well understood, but our analysis will not rely on their details. Our goal is to analyze the domain walls which arise from considering M5-branes wrapping the blown up 4-cycle in these geometries, which behaves effectively like a string of codimension one from the point of view of the 2+1 extended dimensions.

The basic structure of such a domain wall was outlined in [11]. We will take a closer look at the vacuum on both sides of the domain wall in specific setups listed above. There are two main motivations for carrying out this exercise. One is to diagnose the quantization conditions worked out for the fluxes and charges in these backgrounds. If this was done consistently, the charges and the fluxes should also be quantized accordingly on both sides of the domain wall while preserving appropriate conserved quantities. Since some of the ingredients for quantizing the fluxes involved subtle issues [3, 5, 7, 12] such as Freed-Witten [13] and Pontryagin [14] anomalies, it would be a worth while exercise to check the overall consistency of this scheme in the presence of the domain wall. The other motivation stems from the close relation between these domain walls and the tunneling effect which exists on these blown up cones, originally identified in [15]. That a similar phenomenon exists on the Stenzel background was shown recently in [16], and it is not difficult to see similar features also on the  $B_8$  and the LLM backgrounds. These transitions are particularly interesting in the non-BPS context where a candidate meta-stable configuration appears in the probe description of this system. It is interesting to clarify if and when these metastable states are allowed to tunnel to a supersymmetric vacuum.

This note is organized as follows. We begin by explaining the setup and the subtleties for the case of  $B_8$ . Then, we will briefly explain similar issues for the case of Stenzel. Finally, we will examine the case of LLM. As a bonus, we identify a curious stationary brane embedding in the LLM geometry which is interpretable as a probe description of one of these domain

walls.

## 2 Domain wall in mass deformed $B_8$

Let us begin our discussion by considering the case of warped mass deformed  $B_8$ . The  $B_8$  geometry, originally constructed in [17, 18], is an eight dimensional non-compact manifold whose structure is that of a cone over squashed  $S^7$ , deformed by blowing up the tip into an  $S^4$ . The resulting geometry is Asymptotically Conical (AC). The geometry can also be viewed as a spinor bundle over  $S^4$ . This background supports a normalizable anti-self-dual 4-form.<sup>1</sup> When these structures are embedded as part of the solution to eleven dimensional supergravity, the 4-form field strength act as a source to back-react and warp the  $B_8$  geometry. One can add an anti-M2-brane localized in  $B_8$  to further warp the geometry, giving rise to an asymptotically  $AdS_4 \times S^7_{squashed}$  space-time. It is common to take a  $Z_k$  orbifold along the Hopf-fiber of the squashed  $S^7$  in the context of considering this geometry in the context of AdS/CFT-like correspondences.

We should mention that there is another version of eight dimensional space-time, also referred to as the  $B_8$  geometry which asymptotes to a circle fibered over a squashed  $CP^3$  cone. The squashed  $CP^3$  arises naturally as the base of  $U(1)$  fibration of the squashed  $S^7$ . We will refer to these  $B_8$  geometry as the Asymptotically Locally Conical (ALC)  $B_8$  geometries [19, 20].

The AC and ALC  $B_8$  geometries are similar in that they are both non-compact, admit normalizable anti-self-dual 4-forms, exhibit  $spin(7)$  holonomy, and contain an  $S^4$  Lagrangian 4-cycle. However, they exhibit some differences in the asymptotic characterization of charges and fluxes. This is not too surprising in light of the fact that these spaces have different asymptotic geometries. We will therefore consider the cases of AC and ALC asymptotics separately.

### 2.1 Quantization of fluxes on warped $B_8^{AC}$

In this subsection, we will describe the properties of domain wall constructed by wrapping an M5-brane on the  $S^4$  at the tip of asymptotically conical  $B_8/Z_k$  geometry. We begin by reviewing the 11 dimensional supergravity description of the warped  $B_8/Z_k$  background in some detail.

---

<sup>1</sup>For the  $B_8$  background, we are adopting the convention of [3] where the anti-self-dual 4-form and an anti-M2 branes preserve the same supersymmetry.

We start with the Ricci-flat metric of the  $B_8^{AC}$

$$ds_8^2 = \left(1 - \frac{\ell^{10/3}}{r^{10/3}}\right)^{-1} dr^2 + \frac{9}{100} r^2 \left(1 - \frac{\ell^{10/3}}{r^{10/3}}\right) h_i^2 + \frac{9}{20} r^2 d\Omega_4, \quad (2.1)$$

with

$$h_i \equiv \sigma_i - A_{(1)}^i, \quad (2.2)$$

where  $\sigma_i$  are left invariant one-forms on  $SU(2)$ , and  $A_{(1)}^i$  are  $SU(2)$  Yang-Mills instanton on  $S^4$ . For  $\ell = 0$ , this geometry reduces to a cone whose base is a squashed  $S^7$  [21, 22]. The case with finite  $\ell$  corresponds to deforming the tip of this cone so that there is a  $S^4$  of finite radius at  $r = \ell$ .

This geometry admits an anti-self-dual 4-form field strength of the form

$$G_4 = dC_3 \quad (2.3)$$

with

$$C_3 = m (v_1(r)\sigma \wedge X_2 + v_2(r)\sigma \wedge Y_2 + v_3(r)Y_3) + \alpha d\sigma \wedge d\varphi \quad (2.4)$$

and

$$\begin{aligned} v_1(r) &= \frac{\ell^4}{5r^4} + \frac{4\ell^{2/3}}{5r^{2/3}} \\ v_2(r) &= -\frac{\ell^{2/3}}{r^{2/3}} \\ v_3(r) &= \frac{\ell^{2/3}}{r^{2/3}}, \end{aligned} \quad (2.5)$$

where  $\sigma$ ,  $X_2$ ,  $Y_2$ ,  $X_3$ , and  $Y_3$  are differential forms on squashed  $S^7$  defined in [19].

One also can write the 4-form as

$$\begin{aligned} dC &= m \left[ u_1 (ha^2 b dr \wedge \sigma \wedge X_2 \pm c^4 \Omega_4) + u_2 (hbc^2 dr \wedge \sigma \wedge Y_2 \pm a^2 c^2 X_2 \wedge Y_2) \right. \\ &\quad \left. + u_3 (hac^2 dr \wedge Y_3 \mp abc^2 \sigma \wedge X_3) \right] \end{aligned} \quad (2.6)$$

where

$$u_1 = -\frac{800\ell^{2/3}}{27r^{14/3}}, \quad u_2 = -\frac{400\ell^{2/3}}{81r^{14/3}}, \quad u_3 = \frac{400\ell^{2/3}}{81r^{14/3}}. \quad (2.7)$$

The anti-self-dual 4-form sources negative M2 charge. This charge, combined with the charges of additional anti-M2-brane added into the system will give rise to a warp factor, which in the BPS ansatz will take the form

$$\begin{aligned} ds^2 &= H^{-2/3} (-dt^2 + dx_1^2 + dx_2^2) + H^{1/3} ds_8^2 \\ F_4 &= dt \wedge dx_1 \wedge dx_2 \wedge d\tilde{H}^{-1} + mG_4 \end{aligned} \quad (2.8)$$

with  $H$  solving the inhomogeneous condition

$$\nabla^2 H = \frac{1}{2} G_4 \wedge G_4 + (2\pi l_p)^6 Q_2^0 \delta^8(\vec{r}) \quad (2.9)$$

although for simplicity, we will treat the delta function source to be smeared along the  $S^4$ . The parameter  $Q_2^0$  is the brane charge which we will relate to with the various discrete parameters below. Let us point out for now that the brane charge  $Q_2^0$  is not the same as the Page charge  $N$ .

Let us now describe the quantization of parameters in this supergravity background.

First, we quantize the magnitude of the anti-self-dual 4-form as follows. Because  $S^4$  at  $r = \ell$  is a closed surface in this geometry when  $\ell > 0$ , one must impose the quantization of the period of  $G_4$  pulled back on  $S^4$ . This condition reads

$$\int_{S^4} m u_1 c_4 \Omega_4 \Big|_{r=\ell} = -16\pi^2 m = (2\pi l_p)^3 \left( q - \frac{k}{2} \right) \quad (2.10)$$

where  $q$  is an integer. Here, the shift by  $k/2$  is due to the effect of [14] whose presence in the context of the  $B_8$  geometry was argued in [7].

Somewhat less obvious is the quantization of  $\alpha$  in (2.4) which characterizes the torsion class of the 3-form potential restricted to the squashed  $S^7/Z_k$  boundary of the  $B_8^{AC}$ . This is most apparent if one sets  $\ell = 0$  and  $m = 0$  so that we are left with a cone over squashed  $S^7/Z_k$ . Just as was the case for the example in [12], it is somewhat subtle to read off the discrete torsion when  $\ell$  and  $m$  are non-vanishing. One quick way to read off this quantity is to take  $r \rightarrow \infty$  where  $G_4$  goes to zero. This imposes the constraint

$$16\pi^2 \alpha = -(2\pi l_s)^3 g_s \left( l - \frac{k}{2} \right) \quad (2.11)$$

where the shift by  $k/2$  due to the Freed-Witten anomaly has been included since the IIA reduction of squashed  $S^7$  is a squashed  $CP^3$  which contains a non-spin  $CP^2$  homology 4-cycle.

Note, however, that the period of  $G_4$  on any 4-cycles on  $S^7/Z_k$  or its IIA reduction vanishes since the angular component of  $G_4$  decays sufficiently rapidly for large  $r$ . In other words, the D4 Maxwell charge in the IIA reduction vanishes.

We next consider the quantization of M2 charges. It is useful to first consider the case where we set  $N = 0$ . The background will nonetheless carry non-vanishing M2 charge because of the fluxes and the discrete torsion.

The equation for  $G_4$  reads

$$d *_{11} G_4 = \frac{1}{2} G_4 \wedge G_4 . \quad (2.12)$$

Let us denote the flux of  $*G_4$  at  $r = \infty$  and  $r = \ell$  as

$$Q_2^\infty = \frac{1}{(2\pi l_p)^6} \int *G_4 \Big|_{r=\infty} \quad (2.13)$$

$$Q_2^0 = \frac{1}{(2\pi l_p)^6} \int *G_4 \Big|_{r=\ell} . \quad (2.14)$$

The equation of motion (2.12) constrains their difference

$$\begin{aligned} Q_2^\infty - Q_2^0 &= \frac{1}{(2\pi l_p)^6} \int_{\mathcal{M}_8} \frac{1}{2} G_4 \wedge G_4 \\ &= -\frac{\left(q - \frac{k}{2}\right)^2}{2k} \end{aligned} \quad (2.15)$$

which must be negative in order not to break additional supersymmetries because  $G_4$  is anti-self-dual. Since  $q$  is an integer and this quantity can never be set to zero, there is always some non-vanishing M2 charge associated with this background even though we set  $N = 0$ .

Now, in the presence of discrete torsion, one expects

$$Q_2^\infty = -\frac{\left(l - \frac{k}{2}\right)^2}{2k} + \frac{k}{8} = -\frac{l(l-k)}{2k} \quad (2.16)$$

where the additive term  $\frac{k}{8}$  was included to arrange for this contribution to vanish for  $l = 0$ . With this parametrization, the domain wall<sup>2</sup> described in section 3.4.1 of [3] will have the correct induced charge.

This means

$$Q_2^0 = -\frac{l(l-k)}{2k} + \frac{\left(q - \frac{k}{2}\right)^2}{2k} \quad (2.17)$$

which can also be written as

$$Q_2^0 = \frac{k}{8} + \left(l - \frac{k}{2}\right) b_0 + \frac{k}{2} b_0^2 \quad (2.18)$$

where

$$b_0 = -\frac{l-q}{k} \quad (2.19)$$

is the value of  $B$  pulled back on to  $S^2$  in the IIA reduction of  $R^4/Z_k \rightarrow R_+ \times S^2$ .

If one were to add  $N$  units of M2-brane charge at  $r = \ell$  (and smeared along the  $S^4$ ) one finds

$$Q_2^0 = N - \frac{l(l-k)}{2k} + \frac{\left(q - \frac{k}{2}\right)^2}{2k} \quad (2.20)$$

---

<sup>2</sup>This domain wall is localized in the radial direction of the gravity solution and should not be confused with the domain wall on the 2+1 dimensional field theory which is the main subject of this paper.

and

$$Q_2^\infty = N - \frac{l(l-k)}{2k} \quad (2.21)$$

where negative  $N$  corresponds to the branes which preserves the same supersymmetry as the anti-self-dual 4-form field strength  $G_4$ . The condition that these backgrounds are BPS can be expressed simply as

$$Q_2^0 < 0. \quad (2.22)$$

Naive extrapolation of the  $Q_2^0 < 0$  solutions to the  $Q_2^0 > 0$  region introduces a repulsion singularity indicating that the basic ansatz used to construct the solution is breaking down [3]. It is in light of this fact that it is interesting to consider the fate of a probe M2 added to the background with  $Q_2^0 = 0$  which pushes the system just beyond the threshold of supersymmetry breaking. We will come back this issue later in this article.

Since the language and the notation being used here is somewhat different from that of [11], let us provide the map

$$Q_2^\infty = \Phi^{GVW}, \quad Q_2^0 = N^{GVW} \quad (2.23)$$

for the quantity  $\Phi^{GVW}$  and  $N^{GVW}$  introduced in (2.16) of [11]. In the language of [2],  $Q_2^\infty = \Phi^{GVW}$  is the Maxwell charge which is required to remain constant as one crosses the domain wall. For  $k > 1$ , the quantity  $Q_2^0 = N^{GVW}$  is taking on fractional values. These correspond to the brane charge in the language of [2] and are allowed to take on fractional values. For  $k = 1$ ,  $Q_2^0$  actually takes on integer value up to an additive shift by  $k/8$  which, if desired, can be absorbed into the shift of the charge lattice.

## 2.2 Domain wall from wrapped M5 in $B_8^{AC}$

Let us now consider what happens when one wraps an M5-brane on the  $S^4$  Lagrangian cycle at the tip of the deformed  $B_8^{AC}/Z_k$  cone. The  $B_8^{AC}/Z_k$  geometry has the structure of  $R^4/Z_k$  fibered over  $S^4$ , and wrapping an M5-brane on the  $S^4$  will give rise to an object which looks effectively as a codimension one string along the 2+1 extended dimensions.

The issue which one must address is the basic phenomenon that the  $G_4$  flux on the world volume of M5 induces anomalous world volume charge which must be canceled by the correct number of open M2-branes ending on the M5. This is the same basic mechanism which gave rise to open strings ending on the baryon vertex in the construction of [23]. The precise number of open M2-branes required to cancel this anomaly is the quantized period of  $G_4$  on  $S^4$ . This, however, is somewhat problematic since we saw that on  $B_8$ , the flux of  $G_4$  on  $S^4$  can take half integral values when  $k$  is odd. On the first glance, it would appear that half integer unit of open M2 is required to properly cancel the anomaly when  $k$  is odd.



With a little more thought, however, one realizes that this potential problem is already addressed in [11]. The resolution is as follows. If an M5 is wrapped on the  $S^4$ , it must also give rise to a shift in the flux of  $G_4$  by one unit in its dual cycle, which in this case is  $R^4/Z_k$ . Then, on the covering space  $R^4$ , the shift of flux is  $k$ . On the other hand, because the BPS vacuum configuration of  $G_4$  must be anti-self-dual on the covering space,  $q$  must shift by  $k$  unit as one crosses the domain wall.

If the flux of  $G_4$  on  $S^4$  is  $q - k/2$  on one side of the domain wall and  $q + k/2$  on the other side, then their average value is  $q$ . To refer to the average value of the flux seems quite natural from the point of view of taking the thin wall approximation of the domain wall, and is what is employed in [11]. So  $q$  is the number of anomalous open M2 one expects to need in order to cancel the anomalous charge on the world volume of M5 wrapping the  $S^4$ , and this quantity is integer valued.

One can characterize the vacua separated by the domain wall as follows: Suppose on one side of the domain wall, we were provided with the data  $N$ ,  $l$ ,  $k$ , and  $q$  so that  $Q_2^0$  and  $Q_2^\infty$  are as is given in (2.20) and (2.21), respectively. Upon crossing the domain wall,  $q$  shifts to  $q + k$ . In a mean time, the brane charge

$$(Q_2^0)' = N - \frac{l(l-k)}{2k} + \frac{(q + \frac{k}{2})^2}{2k} = Q_2^0 + q \quad (2.24)$$

shifts by  $q$  units. The Maxwell charge  $Q_2^\infty$  is invariant under wall crossing as is expected. The physical picture arising from interpreting the structure of the vacuum on both sides of the domain wall appears to be consistent with the charge quantization conditions outlined in the previous section.

### 2.3 Domain wall from wrapped M5 in $B_8^{ALC}$

In this subsection, we will describe the features of a domain wall arising from wrapping an M5-brane on the blown up  $S^4$  at the tip of asymptotically locally conical  $B_8$ . The  $B_8^{ALC}$  has two physical scales, one associated with the radius of the blown up  $S^4$ , and the one associated with the radius of  $S^1$  at infinity. For a particular numerical value for the dimensionless ratio of these two scales, the  $B_8^{ALC}$  geometry can be viewed as an analytic continuation of another asymptotically locally conical geometry, known as the  $A_8$  geometry, and has a relatively simple analytic form [19]. Generalizing to the case where the ratio of the radius of  $S^4$  and  $S^1$  leads to a somewhat more implicit form of the metric and the flux, but they are known [19, 20]. We will refer to this one parameter family of  $B_8^{ALC}$  geometry as  $B_8^{ALC}(\lambda)$  where  $\lambda$  parametrizes the ratio of the radii as was reviewed in appendix B of [5].

The quantization of fluxes and the interpretation of various distinct notion of charges for

the  $B_8^{ALC}(\lambda)/Z_k$  was described in [5]. It was shown in (5.23) of [5] that the M2 Maxwell charge takes the form

$$Q_2^\infty = N - \frac{l(l-k)}{2k} + \frac{1}{2k} \left( \frac{4C(\lambda)}{25} \right) \left( q - \frac{k}{2} \right)^2 \quad (2.25)$$

where  $C(\lambda)$ , partially illustrated in figure 11.a and figure 12 of [5], takes value ranging from zero to infinity as the ratio of radii of  $S^4$  and  $S^1$  are varied.

Now, imagine wrapping an M5-brane on  $S^4$  in the core region of  $B_8^{ALC}(\lambda)/Z_k$ . Topologically,  $B_8^{ALC}(\lambda)/Z_k$  has the structure, identical to  $B_8^{AC}$ , of an  $R^4/Z_k$  fibered over the  $S^4$  base. As such, just as was the case for the  $B_8^{AC}$ , one expects the insertion of domain wall in the 2+1 extended dimensions by wrapping an M5 brane on the  $S^4$  to cause the flux of four form through  $R^4/Z_k$  and  $S^4$  to jump by one unit across the domain wall. In other words, the value of  $q$  should jump by one across the domain wall.

Now, since the M2 Maxwell charge (2.25) appears to depend explicitly on  $q$ , one may wonder if this jump is causing the M2 Maxwell charge to also jump across the domain wall. This would be contrary to the expectation based on the fact that Maxwell charge is supposed to be a conserved quantity which maps to a parameter defining the theory, as opposed to specifying the vacuum, in the corresponding field theory. This point has caused some confusion, especially to the present author, during the earlier stages of studying this issue.

It turns out that the resolution to this confusion is quite straight forward. There is no need to assume that the value of  $\lambda$  are the same on two sides of the domain wall. In fact, as the flux of  $G_4$  through the  $S^4$  changes, one expects the radius of  $S^4$  to change relative to the radius of  $S^1$ . Since radius of  $S^1$  is fixed at ultra-violet, changes in the radius of  $S^4$  will cause  $\lambda$  to change. Since the M2 Maxwell charge (2.25) must be conserved, we expect  $\lambda$  to adjust itself accordingly. In the case of  $B_8^{AC}$ , this issue did not arise simply because there is no scale relative to which one can measure the radius of the  $S^4$ .

### 3 Domain wall in Stenzel Geometry

As our second example, let us consider the properties of domain walls in Stenzel geometry [24]. Stenzel geometry is a deformation of the cone with the base  $V_{5,2}$  known as the Steifel manifold, by blowing up an  $S^4$ . The embedding of the warped deformed Stenzel geometry into string theory was first considered in [25] and further elaborated in the context of AdS/CFT correspondence in [8]. This geometry can also be viewed as a contangent bundle over  $S^4$ . Asymptotically, Stenzel geometry has the structure of a cone. One difference between this case and the  $B_8^{AC}$  is that the  $Z_k$  orbifold along the  $U(1)_b$  isometry of the

asymptotic  $V_{5,2}$  does not act freely on the  $S^4$ . Nonetheless, one can consider wrapping an M5-brane on the  $S^4$  in this background and interpret it as a domain wall, as was done in [16]. This construction was also treated as the prototype in [11].

The quantization and computation of various charges for the Stenzel geometry was carried out in [12]. We follow the convention of [12] in setting up the geometry and the flux. The M2 brane and Maxwell charge was found in (3.16) and (3.18) of [12] to take the form

$$Q_2^0 = N - \frac{l(l-2k)}{4k} - \frac{kq^2}{4} \quad (3.1)$$

and

$$Q_2^\infty = N - \frac{l(l-2k)}{4k} . \quad (3.2)$$

The parameter  $q$  is the integer quantized period of the self-dual 4-form through the  $S^4$ . There are no anomalous shift by  $k/2$  as was the case for the  $B_8$  in the previous section. Since the four form is taken to be self-dual, the condition for supersymmetry is for  $Q_2^0$  to be positive.

The issue we wish to address now is whether these expressions are compatible with the expected properties of the embedding of the M5-branes.

The first issue we need to establish is the extent to which we expect the  $q$  to shift across the domain wall. For concreteness, let us concentrate on the case where  $k = 1$  so that we can ignore the effects of orbifolding by  $Z_k$ . Then, we do not expect fractional brane sources to exist in the background.

Now consider shifting  $q$  by one in (3.1). This gives rise to

$$\Delta Q_2^0 = -\frac{(q+1)^2}{4} + \frac{q^2}{4} = -\left(\frac{q}{2} + \frac{1}{4}\right) \quad (3.3)$$

which is not integral. Note, however, that if one shifts  $q$  by 2, then the shift in (3.1)

$$\Delta Q_2^0 = -\frac{(q+2)^2}{4} + \frac{q^2}{4} = -(q+1) \quad (3.4)$$

is integral. This turns out to make sense. For one unit self-dual 4-form  $G_4$  normalized so that

$$\int_{S^4} G_4 = 1 \quad (3.5)$$

we have

$$\int_{R^4} G_4 = \frac{1}{2} \quad (3.6)$$

as can be seen in (3.9) and (3.10) of [12] and originally computed in (5.79) and (5.85) of [8]. This means that if crossing the domain wall made by wrapping an M5 on  $S^4$  were to cause the flux of  $G_4$  on the dual  $R^4$  cycle to jump by one unit, then the flux of  $G_4$  through  $S^4$  must

also jump by two units. Shifting  $q$  by two is precisely what we found as giving rise to an integral shift in (3.1). Furthermore, if the value of  $q$  is shifting by 2 units, then taking the average of flux though  $S^4$  on both sides of the domain wall gives

$$\frac{1}{2}((q+2) + q) = q + 1 \quad (3.7)$$

which is precisely the shift found in (3.1) up to a sign which sets the orientation of the M5-brane. All of the numerical details including the factors of 2 conspire to yield a self-consistent picture.

In the case of the  $Z_k$  orbifold, one can work in the covering space for which the shift of the brane charge now reads

$$k \times \Delta Q_2^0 = -k^2(q+1) . \quad (3.8)$$

If before crossing the domain wall the flux of  $G_4$  on  $S^4$  in the covering space is  $kq$ , and on the other side the same flux had jumped to  $k(q+2)$ , then their average is  $k(q+1)$ , and insertion of  $k$  M5-brane will induce precisely  $k^2(q+1)$  units of open M2 charge from the world volume anomaly mechanism, once again giving rise to a consistent picture.

We are therefore finding that the quantization of fluxes and charges which were prescribed in [12] is consistent with the interpretation of M5-branes wrapped on  $S^4$  as domain walls interpolating between consistent supersymmetric vacua in the holographic description of these 2+1 dimensional field theories.

## 4 Domain wall in Lin Lunin Maldacena background

As a final example, we will consider the properties of domain walls arising from M5-branes wrapping 4-cycles in the asymptotically  $AdS_4 \times S^7$  geometry of Lin, Lunin, and Maldacena [9]. There are few features which makes this example different from the examples considered earlier in this article. The most important difference perhaps is the fact that the Lagrangian of the conjectured field theory dual of this geometry is known very explicitly [10], providing opportunities to carry out many detailed comparisons.

### 4.1 Review of the LLM background

Let us begin by reviewing some of the basic features of the LLM solution, which was originally developed to characterize the solutions of type IIB supergravity which were asymptotically  $AdS_5 \times S^5$ . They were able to classify these solutions in terms of a diagram consisting of a flat two dimensional plane, colored in patches by one of two colors, say black and white. In the case where the region covered by one of the colors have finite area, the diagram corresponds

to a specific type IIB geometry asymptoting to  $AdS_5 \times S^5$  whose radius is proportional to the area of the patch in the two dimensional plane.

One can also construct a family of asymptotically pp-wave geometries by taking the Penrose limit of the family of asymptotically  $AdS_5 \times S^5$  geometries described above. These solutions are characterized by the same two dimensional diagram, but with the colored regions exhibiting translation symmetry in one of the coordinates, so that the colored region look somewhat like a bar-code. The transitionally invariant direction along the bar-code diagram corresponds to an isometric direction in the IIB geometry it is representing. One can therefore construct a family of solutions to 11-dimensional supergravity by compactifying that dimension, T-dualizing along this coordinate, and lifting to M-theory. These solutions are characterized by a one dimensional strip like the one illustrated in figure 1.A. It is customary to give the strip a finite width to make it easier to read. If the strip is such that it asymptotes to a solid black region on one end and a solid white region on the other, the 11 dimensional supergravity solution will be asymptotically  $AdS_4 \times S^7$ . To avoid complicating the discussion, we will restrict to the case where we do not perform any  $Z_k$  orbifolding, i.e. set  $k = 1$ . A nice discussion of these structures in the case of  $k > 1$  can be found in [26].

The direction along the strip parametrized by coordinate  $x$  is embedded as one of the coordinates of the M-theory background. In addition, there are semi-infinite coordinates  $y$  and the coordinates of two 3-spheres  $S^3$ ,  $\tilde{S}^3$  parametrizing the 8 dimensions transverse to  $R^{1,2}$ , whose detailed form can be found in (2.33)–(2.35) of [9]. In this construction, the black and white regions correspond to the points in  $x$  where  $S^3$  or  $\tilde{S}^3$ , respectively, shrinks to zero size as  $y \rightarrow 0$ .

This implies that there is an abundance of 4-cycles in this geometry. Any segment straddling a black or white region embedded in the  $xy$  plane whose endpoints are constrained to have  $y = 0$  will define a 4-sphere of the form  $S^4 = I \times S^3$  or  $S^4 = I \times \tilde{S}^3$ . So these 4-cycles are in one to one correspondence with the finite size strips labeled A1, B1, ... B3 in figure 1.A. These 4-cycles link, as is also illustrated in figure 1.A. The semi-infinite black and white strips labeled as B0 and A4 in figure 1.A defines a 4-cycle which is non-compact.

The flux of M-theory 4-form though these 4-cycles are quantized. This turns out to force the length of the colored strip to be quantized as well. It was found in (2.24) of [26] that the length of the finite size strips are integer multiples of

$$\frac{(2\pi l_p)^3 \mu_0}{(2\pi)^2} \quad (4.1)$$

where  $\mu_0$  is a scale parameter included explicitly in the presentation of LLM solution in (2.11) of [26] which we find to be convenient for comparing supergravity and field theory computations.

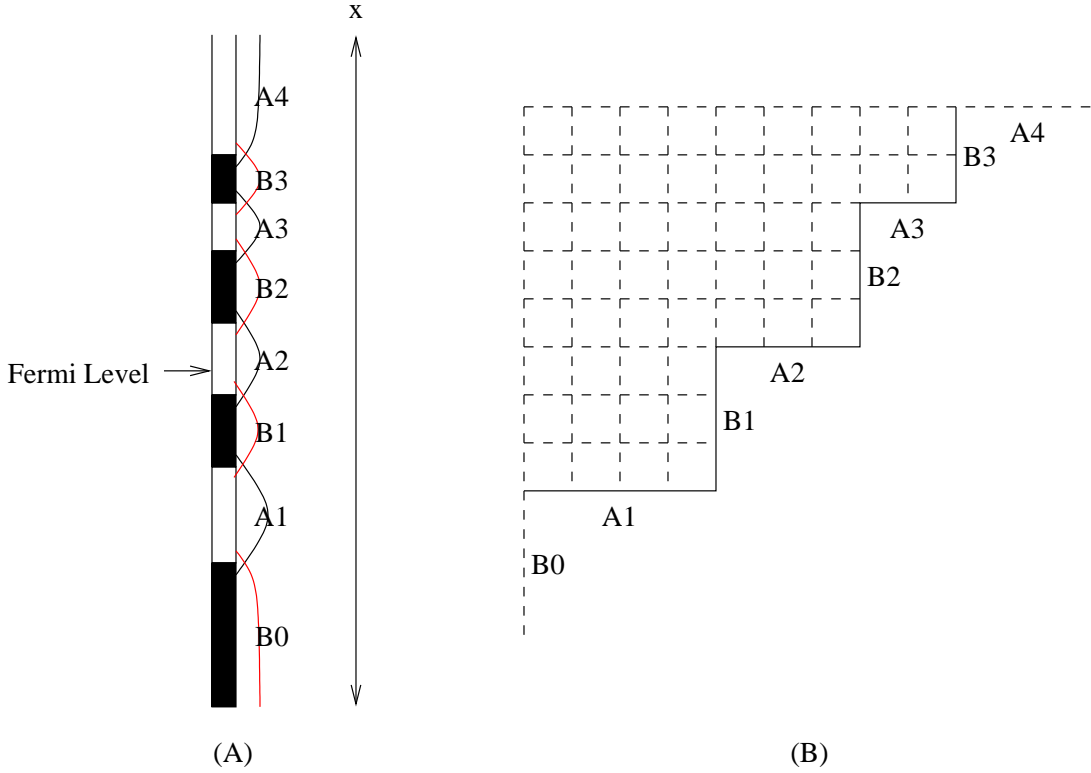


Figure 1: (A) A typical bubble diagram for the asymptotically  $AdS_4 \times S^7$  LLM geometry, and (B) the corresponding Young diagram.

As long as the two semi-infinite strips, i.e. B0 and A4 in figure 1.A is of different color, one can define the Fermi-level which is the point along the strip where the areas, of the black region above and of the white region below this point, are the same. If the black and white regions are interpreted as denoting occupied and unoccupied states in a degenerate fermi-gas system, this point would in fact be interpretable as the Fermi-level of this system. One can also consider the total energy carried by the fermion and hole excitations above the ground state of this degenerate fermi-gas. This quantity turns out to correspond to the radius of the asymptotic  $AdS_4 \times S^7$  region. In other words, it is encoding the Maxwell charge. Transition between different vacua of the same UV field theory corresponds to transition between different excited fermion occupation states with the same total energy.

Another useful way to represent the black and white strip is in the form of a Young tableaux where the white and black strips are mapped to horizontal and vertical edges of the tableaux, respectively, as is illustrated in figure 1.B. In this diagram, the number of boxes correspond to the total energy of the fermi gas and therefore the Maxwell charge of the supergravity background. Young tableaux corresponding to different vacua of the same

theory will therefore have the same number of boxes.

The field theory counterpart of these vacua is also well understood. The candidate dual field theory is the mass deformed ABJM theory constructed by [10, 27]. In order to be completely specific about the conventions including the normalization of the mass parameter  $\mu$ , we will consistently use the model written down in (2.1) and (2.2) [10].<sup>3</sup> It was pointed out by various authors including [10, 26] that the classical vacua of the mass deformed ABJM model can be expressed in a block-diagonalized form illustrated in figure 1 of [26], where each of the sub-blocks are either of dimension  $n \times (n + 1)$ , or of  $(n + 1) \times n$ . The  $n$ 's are allowed to take on any non-negative integer values, and it is not difficult to derive a relation from the fact that these sub-blocks should fit inside an  $N \times N$  matrices that

$$\sum_{n=0}^{\infty} \left( n + \frac{1}{2} \right) (N_n + N'_n) = N, \quad \sum_{n=0}^{\infty} N_n = \sum_{n=0}^{\infty} N'_n. \quad (4.2)$$

In other words,  $N_n$  and  $N'_n$  are naturally interpretable as particle occupation numbers. They will correspond perfectly with the fermion occupation numbers illustrated in figure 1.A if the values of  $N_n$ 's and  $N'_n$ 's are constrained to only take values 0 or 1. (This statement has a simple generalization for the case  $k > 1$  [26].) From the field theory point of view, however,  $N_n$ 's and  $N'_n$ 's can take arbitrary integer values and still give rise to a solution to the equation of motion. This mismatch is one of the subtle unresolved issues in the holographic duality of this system. The general consensus is that the classical vacua with  $N_n$ 's and  $N'_n$ 's taking values larger than one do not exist as a state in the quantum theory. Computation of the Witten index appears to confirm this picture [28]. At the moment, we are missing the understanding of the precise mechanism which destabilizes or lifts these classical vacua which allegedly do not exist at the quantum level. In the remainder of this article, we will adopt the point of view that these states with  $N_n$ 's and  $N'_n$ 's taking values greater than one are indeed absent in the quantum theory.

## 4.2 Domain wall from wrapped M5-branes

Let us now discuss the properties of domain walls constructed by wrapping an M5 branes in one of the many 4-cycles which exists in the LLM background. To be more concrete, let us consider a specific simple configuration of the bubble diagram illustrated in figure 2.A. This is the minimal setup for our purposes. A seemingly smaller setup consisting only of black

---

<sup>3</sup>We warn the reader that appendix C.1 of [10] appears to deviate from their own conventions by various factors of  $\pi/k$ . Appendix C.2, on the other hand, appears to be consistent with the rest of their article. We suspect the unexpected factors of  $\pi/k$  which can be seen, for example, by comparing their expressions for  $V$  at the end of section C.1 and C.2, is a typo.

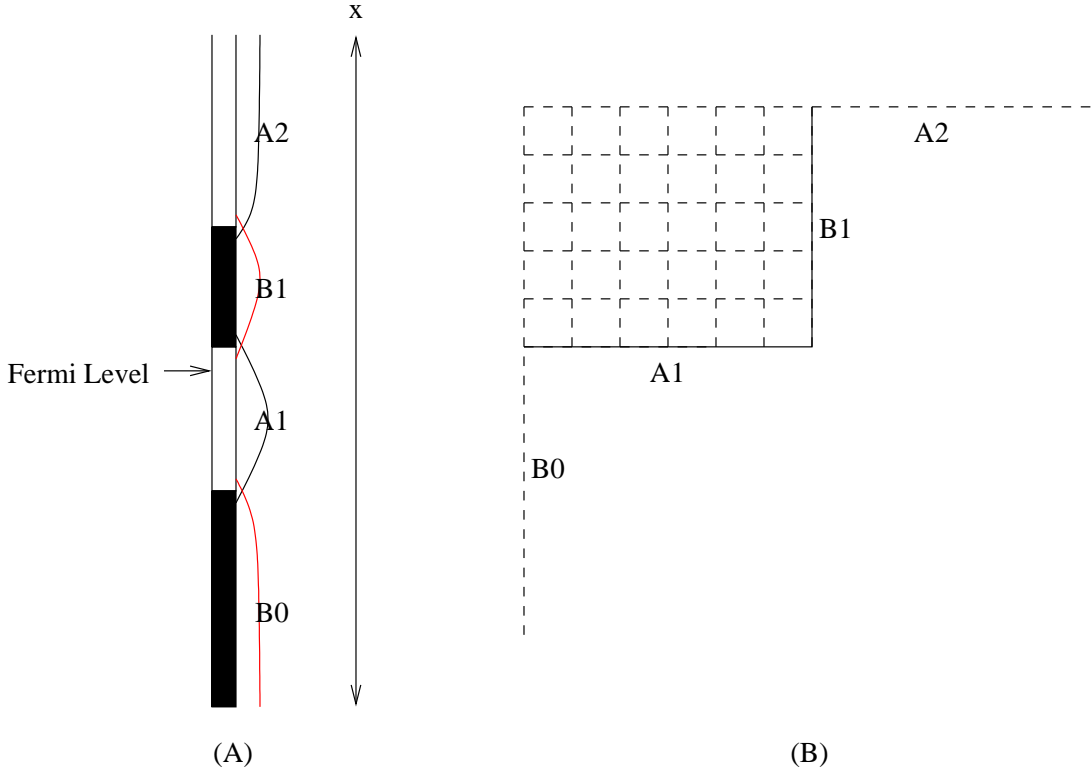


Figure 2: A simple configuration of the asymptotically  $AdS_4 \times S^7$  LLM bubble geometry with compact 4-cycles A1 and B1, and the corresponding Young diagram.

and white semi-infinite strips with no islands have vanishing  $N$  and is a singular geometry.<sup>4</sup>

Let us now imagine wrapping an M5-brane around the 4-cycle labeled as B1 in figure 2. The cycles dual to B1 which links to it are easy to identify. They are the cycles A1 and A2. If as a result of the M5 wrapping B1 the flux though A1 and A2 decrease, and increase, by one, respectively, we will arrive at a new configuration illustrated in figure 3.

Unlike in the cases of  $B_8$  and Stenzel manifolds, the background 4-form flux is not self-dual. Shifting the flux though A1 and A2 as we here do not force the flux of B1 to also shift. The configuration illustrated in figure 3 corresponds to a perfectly sensible background.

One issue with the configuration of 3, however, is the fact that the number of boxes is not the same as the one from figure 2. This is because we have yet to account for the open M2 branes which need to terminate on the M5 to cancel the anomalous world volume charge. That number,  $n_{B1}$ , corresponding to the flux though B1, is precisely the number of boxes deleted in figure 3.

---

<sup>4</sup>One could also consider a configuration from appendix D of [9] but that would destroy the  $AdS_4 \times S^7$  asymptotics and force us to scale  $N$  to infinity.



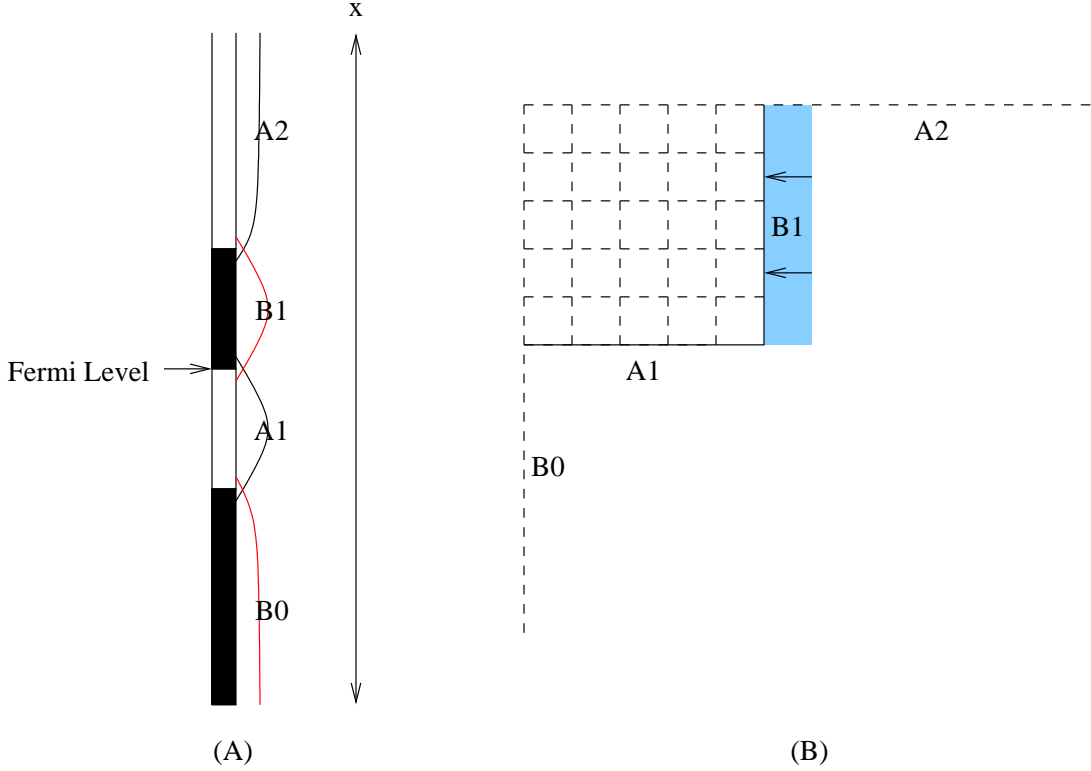


Figure 3: Effects of the back reaction of wrapping an M5-brane on B1.

There are variety of ways in which the  $n_{B1}$  boxes can be added back to the configuration of figure 3. One example is illustrated in figure 4. The point of this configuration is that in the limit where the number of rows and column represented by the flux through A1 and B1 are large, the  $n_{B1}$  additional boxes can be realized in the gravity description effectively in a probe description. Generally, long strips have good gravity descriptions and small strips have good brane probe descriptions [9]. In the configuration illustrated in figure 4, the  $n_{B1}$  M2 branes have merged to form a dielectric M5-brane wrapping a  $S^3$  section of the A2 4-cycle.

### 4.3 Another brane probe domain wall

Before ending this section, let us also describe another curious brane configuration in the LLM geometry which also describes a domain wall interpolating between two distinct vacua.

As a starting point, we recall the fact, originally shown in [26], that an M2 brane probe localized in the 8 dimensions parametrized by  $x$ ,  $y$ ,  $S^3$ , and  $\tilde{S}^3$  are attracted toward  $y = 0$  with points on  $x$  which correspond to the *concave* corners of the Young diagram, i.e. where B0 meets A1, B1 meets A2, etc. Anti-M2 branes, on the other hand, are attracted to  $y = 0$

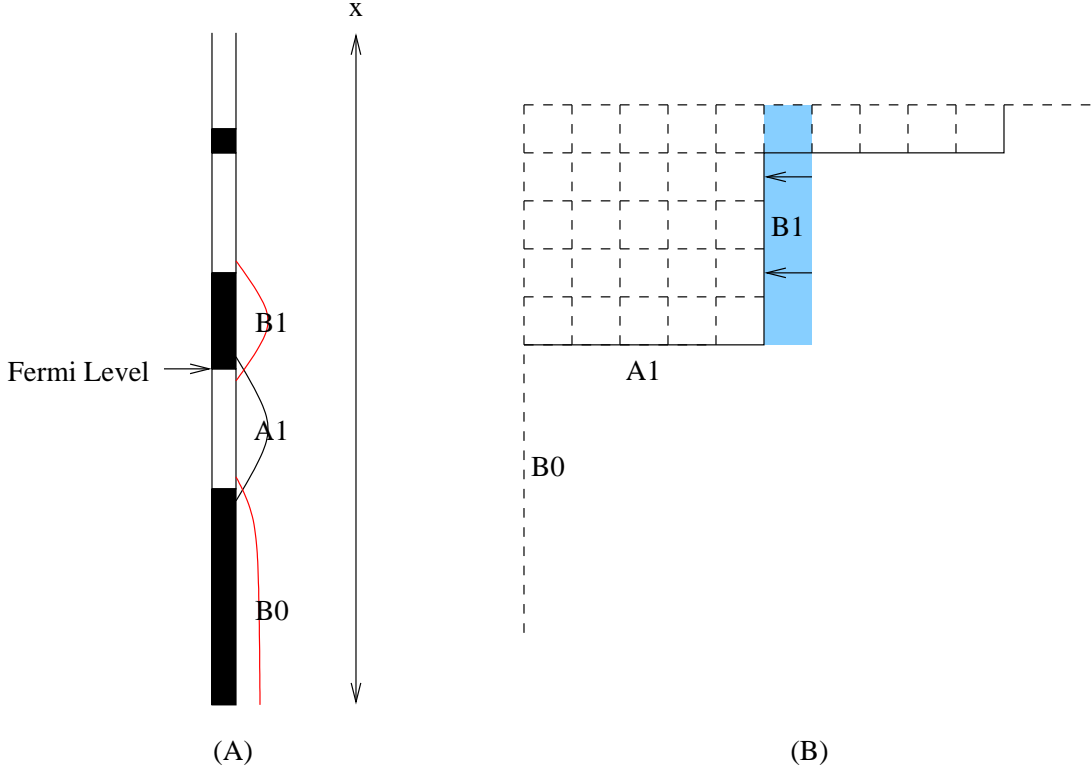


Figure 4: Effects of back reaction of wrapping the M5 on B1 and the  $n_{B1}$  open M2 needed to cancel the induced charges on M5 along B1. There are many possible ways to arrange the M2. This particular configuration allows the M2's, which combine to form a dielectric M5, to be treated in the brane probe approximation.

with points on  $x$  corresponding to the *convex* corners. At these attractor points, the effective tension of the probe branes are degenerate and saturates the BPS condition. It may seem counter-intuitive for an anti M2-brane to coexist as a BPS state in the presence of M2-branes, but such a configuration is made possible by the presence of fluxes.

The effective potential experienced by the M2 and anti M2-brane probes is illustrated in figure 5 of [26]. In light of the degenerate minima, say, for the M2 branes corresponding to the convex corners of the Young diagram, it is natural to contemplate constructing a kink-like embedding of M2-branes interpolating between two of these minima.

To set up this exercise, we need to write down the world volume action for the M2-branes embedded into this background. Alternatively, since we expect the domain wall to be transitionally invariant in one of the extended dimensions of  $R^{1,2}$ , say  $z_2$ , we can compactify this dimension and reduce to IIA description where the M2 becomes a fundamental string. We can then analyze the Nambu-action for the fundamental string. It would be straightfor-

ward to lift the configuration of fundamental string that we find in type IIA description to M-theory.

A useful place to read off the IIA background is (D.1) of [9]. We will also scale in the parameter  $\mu_0$  following [26]. If one wishes to consider an embedding of fundamental string in the  $x$  direction as  $z_1$  is varied, one can parametrize the shape of the string by  $x(z_1)$ . The Nambu action then takes the form

$$L = \left[ e^{2\phi} \sqrt{1 + e^{-2\phi} h^2 \left( \frac{dx}{dz_1} \right)^2} + B \right] \quad (4.3)$$

where  $h$ ,  $B$ , and  $\phi$  are functions of  $x$ .  $B$  is an abbreviation for the  $B_{tz_1}$  component of the NSNS 2-form. Now, since the action does not depend explicitly on  $z$ , one can infer the conservation of the Hamiltonian

$$H = \frac{\partial L}{\partial \left( \frac{\partial L}{\partial z_1} \right)} - L = B - \frac{e^{2\phi}}{\sqrt{1 + e^{-2\phi} h^2 x'(z_1)^2}}. \quad (4.4)$$

In order to impose the asymptotic behavior that  $x'(z_1) = 0$  at  $z_1 = \pm\infty$ , we set  $H = 0$ . Then, we find

$$x'(z_1) = \pm \frac{\sqrt{e^{2\phi}(e^{2\phi} - B)(e^{2\phi} + B)}}{Bh} \quad (4.5)$$

where right hand side is a function of  $x$ . One can therefore find  $z_1(x)$  by integrating this equation.

When this expression is substituted back into the action, it simplifies dramatically to

$$T \int dz_1 L = T \int dz_1 \mu_0 x'(z_1) = T \mu_0 \int dx, \quad T = \frac{1}{2\pi l_s^2}. \quad (4.6)$$

In other words, the tension (which is a mass in the IIA description) is directly proportional to the extent to which the domain wall (particle) is extended in the  $x$  direction. When applied to the configuration like the one illustrated in figure 2.A, one finds that  $x'(z_1)$  indeed vanishes at all  $x$ 's at all of the interfaces of the black and the white regions. In the case where we set these interfaces at  $x = 0$ ,  $x = 1$ , and  $x = 2$ , the  $x'(z_1)$  as is given in (4.5) as a function of  $x$  looks like what is illustrated in figure 5.

That  $x'(z_1)$  is diverging between the interfaces suggests that the embedding of  $x(z)$  is turning around at that point. Numerically solving for  $x(z_1)$  leads to an embedding illustrated in figure 6.

This picture suggests that in the  $z_1 \rightarrow -\infty$  limit, an M2 and an anti M2-brane probe is embedded on the convex and concave corners of the Young diagram, as we illustrate on the

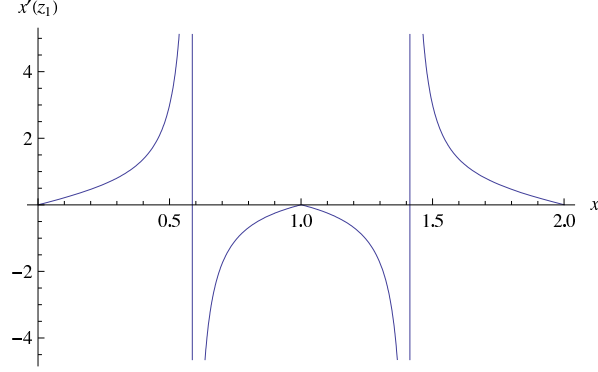


Figure 5: The plot of  $x'(z_1)$  as a function of  $x$  as found in (4.5).

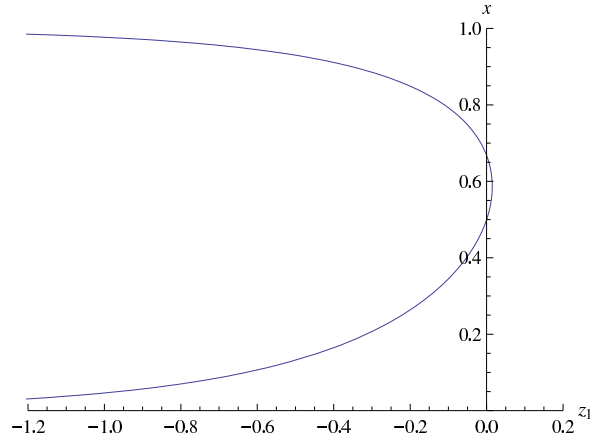


Figure 6: The solution  $x(z_1)$  obtained by numerically solving (4.5).

left side in figure 7. As we increase  $z_1$ , these M2 and anti M2 brane merge and disappear, leaving a configuration illustrated by the second Young diagram in figure 7.

The mass  $T\mu_0\Delta x$  which we computed in type IIA lifts immediately to tension  $\tau = T_{M2}\mu_0\Delta x$  in M-theory. Since  $\Delta x$  is quantized according to (4.1), we infer that the tension of the domain wall is

$$\tau = \frac{\mu_0^2}{2\pi}\Delta n \quad (4.7)$$

where  $\Delta n$  is the quantized length of say the cycle A1 in the configuration of figure 7.

Now, this can be compared against the computation of the domain wall carried out on the field theory side [29]. From reading off the field theory vacua from figure 7, and applying the formula (60) in [29], we read off

$$\tau = \frac{\mu^2}{2\pi}\Delta n . \quad (4.8)$$

So we learn that the two computations match in the scaling with respect to  $\Delta n$ , and are in complete agreement if we identify  $\mu_0$  from the supergravity solution with  $\mu$  as defined in [10].

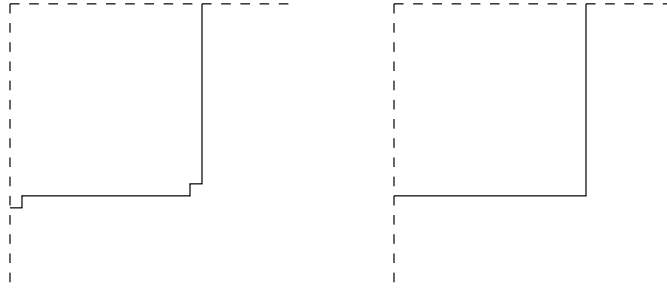


Figure 7: The Young diagram for the vacuum states on two sides of the domain wall represented by the brane embedding illustrated in figure 6.

This identification is consistent with what is reported in [26] based on comparing the masses of the BPS vortices, except for the fact that [26] defines  $\mu$  with an extra power of  $2\pi$  as can be inferred from the form of the superpotential i.e. their (C.18).<sup>5</sup>

Configuration like the one illustrated in figure 6 is quite intriguing. One can introduce a finite mass vortex state connecting the M2 and the anti M2. Such a state, however, can self-annihilate by pushing it toward the domain wall. More generally, a configuration like this appears to offer an interesting laboratory to explore various aspects of brane/anti-brane physics.

## 5 Discussions

In this article, we considered the effects of the back reaction of wrapped M5-brane behaving effectively as a domain wall in the holographic description of certain 2+1 dimensional field theories. These back reaction effects facilitate the shift in the vacuum as one crosses from one side of the domain wall to the other in the gravity description. We studied and confirmed the compatibility between the vacuum structure implied by the back reaction of the domain wall brane and the quantization condition on charges and fluxes intrinsic in these backgrounds. While one expects these issues to ultimately work out consistently, the fact that it actually does so is rather non-trivial in light of the fact that quantization condition of charges and fluxes themselves are quite intricate. Confirming the consistency between the structures of the quantized fluxes and the domain walls can therefore be viewed as a diagnostic.

One interesting issue that this analysis clarifies is the fate of the non-supersymmetric state constructed by starting from the  $B_8$  or the Stenzel background with  $Q_2^0 = 0$  and

---

<sup>5</sup>There appears to be a minor factors of 2 error in (4.7) of [26]. The factor of  $(2\mu_0/\pi T_{M2})$  should instead read  $(\mu_0/2\pi T_{M2})$ . This follows from (2.25) of [26]. We thank Seok Kim for clarification on this point.

adding a supersymmetry breaking M2-brane (or an anti M2-brane). In the cases where the deep IR is superconformal, this has the effect of shifting the brane charge

$$Q_2^0 = N - \frac{l(l-k)}{2k} \quad (5.1)$$

to be slightly negative, and one expects the system not flow to an IR fixed point preserving any supersymmetries. In the case of  $B_8$  and Stenzel, where  $Q_2^0$  has an additional term dependent on  $q$ , one can change the charges carried by the bulk fields to some number of supersymmetry preserving M2-branes at the expense of shifting  $q$ . Such a shift can happen by crossing the domain wall that we considered in this article, or through a tunneling process one constructs by analytically continuing the domain wall solution. Since these M2-branes can annihilate against the supersymmetry breaking M2-branes, the system can relax into the supersymmetric vacua.

The only exception to this scenario is the case where the magnitude of the self-dual 4-form (or the anti-self-dual 4-form) is vanishing. In this case, there are no charges carried by the bulk fields that one can harvest using the domain wall/tunneling transition to annihilate the supersymmetry breaking M2-brane. Furthermore, if we set the 4-form and the brane charge to zero, then we are also setting all of the sources of warping, i.e. the Maxwell charge, to zero. So clearly, this is a degenerate case.

It is also interesting to see a consistent picture emerging also in the case of the correspondence between LLM and the mass deformed ABJM model. What makes this example special is the fact that the field theory side of the correspondence is better understood than the other examples. One curious feature of the LLM geometry on the gravity side, in contrast to the other examples, is the fact that in the LLM case, the topology of the 4-cycles are the consequence of the 4-form fluxes, whereas in the other examples the 4-cycles existed independently of turning on the flux. There may be more important lessons pertaining to dynamical topology changing processes in string theory related to this observation.

Another interesting feature of these mass deformed theories is that they appear to support a set of degenerate vacua. It would be useful to identify a set of physical observables which can be used as a way to discriminate between these degenerate vacua. On the field theory side, the expectation value of the superpotential is a partial probe of this issue. A natural candidate dual of this observable is the superpotential of Gukov, Vafa, and Witten [11]. These superpotentials should also be useful for computing the tension of domain walls. It would be interesting to demonstrate a more detailed correspondence between the superpotential of the field theory side and the GVW superpotential, especially for the case of LLM/mass deformed ABJM theory where a lot is known already on the field theory side.

## Acknowledgements

We would like to thank Ofer Aharony, Shinji Hirano, and Peter Ouyang for collaboration on related issues and for discussions at the early stage of this work and Horatiu Nastase for a discussion on fuzzy 3 spheres and mass deformed ABJM theories. We also thank Seok Kim, Igor Klebanov, Oleg Lunin, Peter Ouyang, and Diego Trancanelli for helpful comments and discussions. This work was supported in part by the DOE grant DE-FG02-95ER40896.

## References

- [1] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, “ $\mathcal{N} = 6$  superconformal Chern-Simons-matter theories, M2-branes and their gravity duals,” *JHEP* **0810** (2008) 091, 0806.1218.
- [2] D. Marolf, “Chern-Simons terms and the three notions of charge,” [hep-th/0006117](#).
- [3] O. Aharony, A. Hashimoto, S. Hirano, and P. Ouyang, “D-brane charges in gravitational duals of 2+1 dimensional gauge theories and duality cascades,” *JHEP* **01** (2010) 072, 0906.2390.
- [4] A. Hashimoto and P. Ouyang, “Supergravity dual of Chern-Simons Yang-Mills theory with  $\mathcal{N} = 6, 8$  superconformal IR fixed point,” *JHEP* **10** (2008) 057, 0807.1500.
- [5] A. Hashimoto, S. Hirano, and P. Ouyang, “Branes and fluxes in special holonomy manifolds and cascading field theories,” [1004.0903](#).
- [6] I. Bena, G. Giecold, and N. Halmagyi, “The backreaction of anti-M2 branes on a warped Stenzel space,” *JHEP* **04** (2011) 120, 1011.2195.
- [7] S. Gukov and J. Sparks, “M-theory on  $Spin(7)$  manifolds. I,” *Nucl. Phys.* **B625** (2002) 3–69, [hep-th/0109025](#).
- [8] D. Martelli and J. Sparks, “ $AdS_4/CFT_3$  duals from M2-branes at hypersurface singularities and their deformations,” *JHEP* **12** (2009) 017, 0909.2036.
- [9] H. Lin, O. Lunin, and J. M. Maldacena, “Bubbling AdS space and 1/2 BPS geometries,” *JHEP* **0410** (2004) 025, [hep-th/0409174](#).
- [10] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk, and H. Verlinde, “A massive study of M2-brane proposals,” *JHEP* **0809** (2008) 113, 0807.1074.

- [11] S. Gukov, C. Vafa, and E. Witten, “CFT’s from Calabi-Yau four-folds,” *Nucl. Phys.* **B584** (2000) 69–108, [hep-th/9906070](#).
- [12] A. Hashimoto and P. Ouyang, “Quantization of charges and fluxes in warped Stenzel geometry,” [1104.3517](#).
- [13] D. S. Freed and E. Witten, “Anomalies in string theory with D-branes,” [hep-th/9907189](#).
- [14] E. Witten, “On flux quantization in M-theory and the effective action,” *J. Geom. Phys.* **22** (1997) 1–13, [hep-th/9609122](#).
- [15] S. Kachru, J. Pearson, and H. L. Verlinde, “Brane/Flux annihilation and the string dual of a non-supersymmetric field theory,” *JHEP* **06** (2002) 021, [hep-th/0112197](#).
- [16] I. R. Klebanov and S. S. Pufu, “M-Branes and metastable states,” [1006.3587](#).
- [17] G. W. Gibbons, D. N. Page, and C. N. Pope, “Einstein metrics on  $S^3$ ,  $R^3$  and  $R^4$  bundles,” *Commun. Math. Phys.* **127** (1990) 529.
- [18] R. Bryant and S. Salamon, “On the construction of some complete metrics with exceptional holonomy,” *Duke Math. J.* **58** (1989) 829.
- [19] M. Cvetič, G. W. Gibbons, H. Lu, and C. N. Pope, “New complete non-compact  $Spin(7)$  manifolds,” *Nucl. Phys.* **B620** (2002) 29–54, [hep-th/0103155](#).
- [20] M. Cvetič, G. W. Gibbons, H. Lu, and C. N. Pope, “Cohomogeneity one manifolds of  $Spin(7)$  and  $G_2$  holonomy,” *Phys. Rev.* **D65** (2002) 106004, [hep-th/0108245](#).
- [21] M. J. Duff, B. E. W. Nilsson, and C. N. Pope, “Spontaneous supersymmetry breaking by the squashed seven sphere,” *Phys. Rev. Lett.* **50** (1983) 2043.
- [22] M. J. Duff, B. E. W. Nilsson, and C. N. Pope, “Kaluza-Klein supergravity,” *Phys. Rept.* **130** (1986) 1–142.
- [23] E. Witten, “Baryons and branes in anti de Sitter space,” *JHEP* **07** (1998) 006, [hep-th/9805112](#).
- [24] M. Stenzel, “Ricci-flat metrics on the complexification of a compact rank one symmetric space,” *Manuscr. Math.* **80** (1993) 151–163.
- [25] M. Cvetič, G. W. Gibbons, H. Lu, and C. N. Pope, “Ricci-flat metrics, harmonic forms and brane resolutions,” *Commun. Math. Phys.* **232** (2003) 457–500, [hep-th/0012011](#).



- [26] S. Cheon, H.-C. Kim, and S. Kim, “Holography of mass-deformed M2-branes,”  
1101.1101.
- [27] K. Hosomichi, K.-M. Lee, S. Lee, S. Lee, and J. Park, “ $\mathcal{N} = 5, 6$  superconformal  
Chern-Simons theories and M2-branes on Orbifolds,” *JHEP* **0809** (2008) 002,  
0806.4977.
- [28] H.-C. Kim and S. Kim, “Supersymmetric vacua of mass-deformed M2-brane theory,”  
*Nucl.Phys.* **B839** (2010) 96–111, 1001.3153.
- [29] K. Hanaki and H. Lin, “M2-M5 systems in  $\mathcal{N} = 6$  Chern-Simons theory,” *JHEP* **0809**  
(2008) 067, 0807.2074.